

Ramps and subsets

The left half of the table below shows ramps, i.e. sequences $r_0 r_1 r_2 r_3$ with $0 \leq r_0 \leq r_1 \leq r_2 \leq r_3 \leq 2$. The right half shows subsets of size 4 in $\{0, 1, 2, 3, 4, 5\}$. Each ramp gives a subset $\{r_0, r_1 + 1, r_2 + 2, r_3 + 3\}$.

0000	0001	0002	$\{0, 1, 2, 3\}$	$\{0, 1, 2, 4\}$	$\{0, 1, 2, 5\}$
0011	0012	0022	$\{0, 1, 3, 4\}$	$\{0, 1, 3, 5\}$	$\{0, 1, 4, 5\}$
0111	0112	0122	$\{0, 2, 3, 4\}$	$\{0, 2, 3, 5\}$	$\{0, 2, 4, 5\}$
0222	1111	1112	$\{0, 3, 4, 5\}$	$\{1, 2, 3, 4\}$	$\{1, 2, 3, 5\}$
1122	1222	2222	$\{1, 2, 4, 5\}$	$\{1, 3, 4, 5\}$	$\{2, 3, 4, 5\}$

Finding the next ramp

We are looking for ramps $r_0r_1r_2r_3$ with $0 \leq r_0 \leq r_1 \leq r_2 \leq r_3 \leq 2$

- ▶ Start with 0000.
- ▶ The last entry in 0000 is less than 2 so we can increase it to give 0001
- ▶ The last entry in 0001 is less than 2 so we can increase it to give 0002
- ▶ Working backwards, the first entry in 0002 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 0011.
- ▶ The last entry in 0011 is less than 2 so we can increase it to give 0012
- ▶ Working backwards, the first entry in 0012 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0022.
- ▶ Working backwards, the first entry in 0022 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 0111.
- ▶ The last entry in 0111 is less than 2 so we can increase it to give 0112
- ▶ Working backwards, the first entry in 0112 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0122.
- ▶ Working backwards, the first entry in 0122 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0222.
- ▶ Working backwards, the first entry in 0222 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 1111.

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- ▶ Working backwards, the first entry in 0022 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 0111.
- ▶ The last entry in 0111 is less than 2 so we can increase it to give 0112
- ▶ Working backwards, the first entry in 0112 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0122.
- ▶ Working backwards, the first entry in 0122 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0222.
- ▶ Working backwards, the first entry in 0222 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 1111.

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- ▶ Working backwards, the first entry in 0022 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 0111.
- ▶ The last entry in 0111 is less than 2 so we can increase it to give 0112
- ▶ Working backwards, the first entry in 0112 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0122.
- ▶ Working backwards, the first entry in 0122 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0222.
- ▶ Working backwards, the first entry in 0222 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 1111.

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- ▶ Working backwards, the first entry in 0012 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0022.
- ▶ Working backwards, the first entry in 0022 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 0111.
- ▶ The last entry in 0111 is less than 2 so we can increase it to give 0112
- ▶ Working backwards, the first entry in 0112 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0122.
- ▶ Working backwards, the first entry in 0122 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0222.
- ▶ Working backwards, the first entry in 0222 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 1111.

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- ▶ The last entry in 0011 is less than 2 so we can increase it to give 0012
- ▶ Working backwards, the first entry in 0012 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0022.
- ▶ Working backwards, the first entry in 0022 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 0111.
- ▶ The last entry in 0111 is less than 2 so we can increase it to give 0112
- ▶ Working backwards, the first entry in 0112 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0122.
- ▶ Working backwards, the first entry in 0122 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0222.
- ▶ Working backwards, the first entry in 0222 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 1111.

Finding the next ramp

We are looking for ramps $r_0r_1r_2r_3$ with $0 \leq r_0 \leq r_1 \leq r_2 \leq r_3 \leq 2$

- ▶ Start with 0000.
- ▶ The last entry in 000**0** is less than 2 so we can increase it to give 0001
- ▶ The last entry in 000**1** is less than 2 so we can increase it to give 0002
- ▶ Working backwards, the first entry in 00**0**2 that is < 2 is the indicated **0**. We increase this to 1 and fill the rest of the sequence with 1s giving 0011.
- ▶ The last entry in 001**1** is less than 2 so we can increase it to give 0012
- ▶ Working backwards, the first entry in 00**1**2 that is < 2 is the indicated **1**. We increase this to 2 and fill the rest of the sequence with 2s giving 0022.
- ▶ Working backwards, the first entry in 0**0**22 that is < 2 is the indicated **0**. We increase this to 1 and fill the rest of the sequence with 1s giving 0111.
- ▶ The last entry in 011**1** is less than 2 so we can increase it to give 0112
- ▶ Working backwards, the first entry in 01**1**2 that is < 2 is the indicated **1**. We increase this to 2 and fill the rest of the sequence with 2s giving 0122.
- ▶ Working backwards, the first entry in 0**1**22 that is < 2 is the indicated **1**. We increase this to 2 and fill the rest of the sequence with 2s giving 0222.
- ▶ Working backwards, the first entry in **0**222 that is < 2 is the indicated **0**. We increase this to 1 and fill the rest of the sequence with 1s giving 1111.

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We are looking for ramps $r_0r_1r_2r_3$ with $0 \leq r_0 \leq r_1 \leq r_2 \leq r_3 \leq 2$

- ▶ Start with 0000.
- ▶ The last entry in 0000 is less than 2 so we can increase it to give 0001
- ▶ The last entry in 0001 is less than 2 so we can increase it to give 0002
- ▶ Working backwards, the first entry in 0002 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 0011.
- ▶ The last entry in 0011 is less than 2 so we can increase it to give 0012
- ▶ Working backwards, the first entry in 0012 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0022.
- ▶ Working backwards, the first entry in 0022 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 0111.
- ▶ The last entry in 0111 is less than 2 so we can increase it to give 0112
- ▶ Working backwards, the first entry in 0112 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0122.
- ▶ Working backwards, the first entry in 0122 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0222.
- ▶ Working backwards, the first entry in 0222 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 1111.

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We are looking for ramps $r_0r_1r_2r_3$ with $0 \leq r_0 \leq r_1 \leq r_2 \leq r_3 \leq 2$

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- ▶ Working backwards, the first entry in 00**0**2 that is < 2 is the indicated **0**. We increase this to 1 and fill the rest of the sequence with 1s giving 0011.
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- ▶ The last entry in 011**1** is less than 2 so we can increase it to give 0112
- ▶ Working backwards, the first entry in 01**1**2 that is < 2 is the indicated **1**. We increase this to 2 and fill the rest of the sequence with 2s giving 0122.
- ▶ Working backwards, the first entry in 0**1**22 that is < 2 is the indicated **1**. We increase this to 2 and fill the rest of the sequence with 2s giving 0222.
- ▶ Working backwards, the first entry in 0**2**22 that is < 2 is the indicated **0**. We increase this to 1 and fill the rest of the sequence with 1s giving 1111.

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- ▶ Working backwards, the first entry in 00**1**2 that is < 2 is the indicated **1**. We increase this to 2 and fill the rest of the sequence with 2s giving 0022.
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- ▶ Working backwards, the first entry in 0**1**22 that is < 2 is the indicated **1**. We increase this to 2 and fill the rest of the sequence with 2s giving 0222.
- ▶ Working backwards, the first entry in **0**222 that is < 2 is the indicated **0**. We increase this to 1 and fill the rest of the sequence with 1s giving 1111.

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- ▶ Working backwards, the first entry in 0012 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0022.
- ▶ Working backwards, the first entry in 0022 that is < 2 is the indicated 0. We increase this to 1 and fill the rest of the sequence with 1s giving 0111.
- ▶ The last entry in 0111 is less than 2 so we can increase it to give 0112
- ▶ Working backwards, the first entry in 0112 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0122.
- ▶ Working backwards, the first entry in 0122 that is < 2 is the indicated 1. We increase this to 2 and fill the rest of the sequence with 2s giving 0222.
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