MAS61015 ALGEBRAIC TOPOLOGY — PROBLEM SHEET 15

Please hand in exercises 3 and 4 by the Wednesday lecture of Week 10.

Exercise 1. Recall that the Möbius strip can be defined as

$$M = \{(z,w) \in S^1 \times B^2 \mid w^2/z \in [0,1] \subseteq \mathbb{R} \subseteq \mathbb{C}\}.$$

Show that there is a covering map $p\colon S^1 \times [-1,1] \to M$.

Exercise 2. Let $p\colon X \to Y$ be a covering map. Let Y_0 be a subset of Y, and put $X_0 = p^{-1}(Y_0)$, so we have a restricted map $p_0\colon X_0 \to Y_0$. Give X_0 and Y_0 the subspace topologies inherited from X and Y respectively. Prove that p_0 is also a covering.

Exercise 3. Suppose that $p_0\colon X_0 \to Y_0$ and $p_1\colon X_1 \to Y_1$ are covering maps. Define $p = p_0 \times p_1\colon X_0 \times X_1 \to Y_0 \times Y_1$, so $p(x_0, x_1) = (p_0(x_0), p_1(x_1))$. Show that p is a covering map (with respect to the product topologies on $X_0 \times X_1$ and $Y_0 \times Y_1$).

Exercise 4. Let $T = S^1 \times S^1$ be the torus, and define $p\colon T \to T$ by $p(u,v) = (u^2, v^2)$. Prove that p is a covering. Put $Y = \{(u,v) \in T \mid u = 1 \text{ or } v = 1\}$ and $X = p^{-1}(Y)$. Draw a picture of X, as a finite collection of points and arcs joining them. Draw a similar picture of Y, and annotate your pictures to illustrate the effect of the covering map $p\colon X \to Y$.

Exercise 5. Let $p\colon X \to Y$ be a 1-sheeted covering. Prove that p is a homeomorphism.